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A new spin-2 self-dual model in $D = 2 + 1$

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Abstract: There are three self-dual models of massive particles of helicity +2 (or −2) in $D = 2 + 1$. Each model is of first, second, and third-order in derivatives. Here we derive a new self-dual model of fourth-order, $\mathcal{L}_{SD}^{(4)}$, which follows from the third-order model (linearized topologically massive gravity) via Noether embedment of the linearized Weyl symmetry. In fact, each self-dual model can be obtained from the previous one $\mathcal{L}_{SD}^{(i)} \to \mathcal{L}_{SD}^{(i+1)}$, $i = 1, 2, 3$ by the Noether embedment of an appropriate gauge symmetry, culminating in $\mathcal{L}_{SD}^{(4)}$. The new model may be identified with the linearized version of $\mathcal{L}_{HDTMG} = \epsilon^{\mu\nu\rho}\Gamma^{\epsilon}_{\mu\gamma}\left[\partial_\nu\Gamma^{\gamma}_{\epsilon\rho} + (2/3)\Gamma^{\gamma}_{\nu\delta}\Gamma^{\delta}_{\rho\epsilon}\right]/8m + \sqrt{-g}\left[R_{\mu\nu}R^{\nu\mu} - 3\,R^2/8\right]/2m^2$. We also construct a master action relating the third-order self-dual model to $\mathcal{L}_{SD}^{(4)}$ by means of a mixing term with no particle content which assures spectrum equivalence of $\mathcal{L}_{SD}^{(4)}$ to other lower-order self-dual models despite its pure higher derivative nature and the absence of the Einstein-Hilbert action. The relevant degrees of freedom of $\mathcal{L}_{SD}^{(4)}$ are encoded in a rank-two tensor which is symmetric, traceless and transverse due to trivial (non-dynamic) identities, contrary to other spin-2 self-dual models. We also show that the Noether embedment of the Fierz-Pauli theory leads to the new massive gravity of Bergshoeff, Hohm and Townsend.

Keywords: Duality in Gauge Field Theories, Chern-Simons Theories, Gauge Symmetry

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Contents

1 Introduction

It is known that in $D = 2+1$, massive particles of helicity $+1$ (or -1) can be described either by a second order gauge theory [1], Maxwell-Chern-Simons ($\mathcal{L}_{MCS}(A_\mu)$), or by a first-order nongauge theory [2], self-dual model ($\mathcal{L}_{SD}(f)$). The physical equivalence of both theories can be established via a master action [3] depending on both fields A_μ and f_μ which is obtained from the self-dual model $\mathcal{L}_{SD}(f)$ by adding a mixing term between the fields A_μ and f_μ. Since the mixing term is a pure first-order Chern-Simons term CS_1 with no particle content, the physical equivalence between $\mathcal{L}_{MCS}(A_\mu)$ and $\mathcal{L}_{SD}(f)$ follows trivially. In particular, this explains why the propagator of the MCS theory contains an innocuous (vanishing residue [4]) massless pole besides the physical massive pole present in the self-dual model of [2]. Namely, the non-propagating massless pole is inherited from the pure Chern-Simons term. Alternatively, one can derive the MCS theory out of $\mathcal{L}_{SD}(f)$ via a two steps Noether embedment ,see [5], of the gauge symmetry $\delta_\Lambda f_\mu = \partial_\mu \Lambda$ of the Chern-Simons term present in $\mathcal{L}_{SD}(f)$. Since a couple of parity singlets of opposite helicities $+1$ and -1 can be combined into one parity doublet (Proca theory), one might try to apply the Noether gauge embedment (NGE) procedure directly to the Proca model. Indeed, in the begin of the next section, as an introduction to the rest of the work, we show that in this case we obtain a ("wrong" sign) Maxwell-Podolsky theory which contains a massive physical particle plus a massless ghost in the spectrum. Thus, the NGE procedure, in this case, fails to produce a physical gauge theory. The appearance of ghosts via NGE has been noticed before in [4]. The analogous of the Proca model for spin-2 particles is the Fierz-Pauli theory. In the next section we show that in this case the NGE procedure leads, in $D = 2+1$, precisely to the new massive gravity theory of Bergshoeff, Hohm and Townsend (BHT model henceforth) [6]. Such theory shares the same spectrum of the Fierz-Pauli theory. We explain the difference between the spin-1 and spin-2 cases based on the different particle contents of the Einstein-Hilbert and Maxwell actions.

In the third and main section we apply the NGE procedure to parity singlets of helicity +2. We show that the three known self-dual models of first- ($S_{\rm SD}^{(1)}$), second- ($S_{\rm SD}^{(2)}$) and third-order ($S_{\rm SD}^{(3)}$), see respectively [1, 7, 8], can be related ($S_{\rm SD}^{(1)} \to S_{\rm SD}^{(2)} \to S_{\rm SD}^{(3)}$) via Noether embedment of appropriate gauge symmetries, which is in agreement with the triple master action of [9]. In subsection 3.3, by embedding a linearized Weyl symmetry present in part of $S_{\rm SD}^{(3)}$ (linearized topologically massive gravity (LTMG)) we obtain a previously unknown fourth-order self-dual model ($S_{\rm SD}^{(4)}$) dual to the other self-dual models which completes the sequence of embedment with $S_{\rm SD}^{(3)} \to S_{\rm SD}^{(4)}$. In section 4 we draw our conclusions.

2 Gauge embedment of parity doublets

2.1 The spin-1 case

It is known that massive particles of spin-1 are described in a covariant way by the Proca model:

$$\mathcal{L}_P = -\frac{1}{4}F^{\mu\nu}F_{\mu\nu} - \frac{m^2}{2}A^\mu A_\mu. \tag{2.1}$$

Throughout this work we use, $\mu,\nu = 0,1,2$ and the signature is $\eta_{\mu\nu} = (-,+,+)$. From the equations of motion of (2.1) one derives the transverse condition $\partial_\mu A^\mu = 0$ and the Klein-Gordon equation $(\Box - m^2)A^\mu = 0$. The Lagrangian (2.1) contains a parity doublet of helicities +1 and −1 in $D = 2+1$, for a simple derivation see [10]. The Maxwell term is invariant under the gauge transformation $\delta_\Lambda A_\mu = \partial_\mu\Lambda$ which is broken by the mass term. One might wonder whether there would exist a gauge invariant description of spin-1 massive particles. Let us show how does the Noether gauge embedment procedure [5] work in this case. The gauge variation of the Proca action, $S_P = \int d^3x\, \mathcal{L}_P$, can be written as:

$$\delta_\Lambda S_P = \int d^3x\, K^\nu \partial_\nu \Lambda \tag{2.2}$$

The Euler vector is given by $K^\nu = \delta S_P/\delta A_\nu = (\Box\theta^{\nu\mu} - m^2 g^{\nu\mu})A_\mu$ with $\Box\theta^{\nu\mu} = \Box\eta^{\mu\nu} - \partial^\mu\partial^\nu$. As a first step in the Noether procedure one introduces a compensating auxiliary vector field whose gauge transformation is given by $\delta_\Lambda a_\nu = -\partial_\nu\Lambda$ such that:

$$\begin{aligned}\delta_\Lambda S_1 &\equiv \delta_\Lambda\left(S_P + \int d^3x\, K^\nu a_\nu\right) = \int d^3x\, \delta_\Lambda K^\nu a_\nu \\ &= \int d^3x\, \left(-m^2\partial^\nu\Lambda\, a_\nu\right) = \delta_\Lambda \int d^3x \left(\frac{m^2}{2}a^\nu a_\nu\right).\end{aligned} \tag{2.3}$$

Therefore,

$$\delta_\Lambda S_2 \equiv \delta_\Lambda \int d^3x \left(\mathcal{L}_P + K^\nu a_\nu - \frac{m^2}{2}a^\nu a_\nu\right) = 0 \quad . \tag{2.4}$$

Eliminating the auxiliary field a_μ by means of its equations of motion from the second iterated action S_2 defined above we end up with the higher-order gauge invariant action:

$$S_{\rm inv.} = \int d^3x \left(\mathcal{L}_P + \frac{K^\nu K_\nu}{2m^2} \right) = \frac{1}{4} \int d^3x \, F^{\mu\nu} \left(1 - \frac{\Box}{m^2} \right) F_{\mu\nu} \tag{2.5}$$

The addition of a quadratic term in the Euler vector to the Proca theory guarantees that an arbitrary variation $\delta S_{\rm inv.} = \int d^3x \, K^\nu \left(\delta A_\nu + \delta K_\nu / m^2 \right)$ will vanish at $K_\nu = 0$. So the solutions of the equations of motion of the original Proca theory will be also solutions of the equations of motion of the new action $S_{\rm inv.}$. Thus, the Proca theory is embedded in the gauge theory (2.5) which is the three dimensional analogue of the Podolsky [11] theory but with an opposite overall sign. The equations of motion of $S_{\rm inv.}$, i.e., $\left(\Box - m^2\right) \partial^\mu F_{\mu\nu} = 0$, in the gauge $\partial^\mu A_\mu = 0$, lead to $\Box \left(\Box - m^2\right) A_\mu = 0$. So, besides the expected massive particle we have also a massless mode. The overall sign in (2.5) is such that the massive particle is physical and the massless one is a ghost in agreement with [4]. This is contrary to the Podolsky theory which is known to contain a massless photon and a massive ghost, see comment in [12]. In summary, we have not succeeded in deriving a physical gauge theory for spin 1 particles by a direct embedment of (2.1).[1] This can be better understood from the master action point of view. In the master action approach for massive theories [3, 13, 14], we add to the original non-gauge theory, the Proca model, a mixing term between the dual fields with the desired gauge symmetry such that the highest derivative term of the non-gauge theory is canceled. In the present case we are led to the Master action:

$$S_M(A, \tilde{A}) = \int d^3x \left[-\frac{1}{4} F_{\mu\nu}(A) F^{\mu\nu}(A) - \frac{m^2}{2} A_\mu A^\mu + \frac{1}{4} F_{\mu\nu}(A - \tilde{A}) F^{\mu\nu}(A - \tilde{A}) \right] \tag{2.6}$$

The action (2.6) is invariant under $\delta_\Lambda \tilde{A}_\mu = \partial_\mu \Lambda$. Due to the positive sign in front of the Maxwell-type mixing term the path integral over the non-gauge field A_μ leads to a local theory which is exactly the gauge theory (2.5) with A_μ replaced by the dual gauge field $\tilde{A}_\mu$. On the other hand if we make the shift $\tilde{A}_\mu \to \tilde{A}_\mu + A_\mu$ in (2.6) before any integration we obtain a Proca theory plus a decoupled Maxwell term with "wrong" overall sign which is the origin of the massless ghost in agreement with [4].

2.2 The spin-2 case

In the last subsection, we have obtained a higher order Maxwell-Podolski-type model with a ghost. Here we will see that the same procedure applied to the Fierz-Pauli theory (spin-2 analogue of Maxwell-Proca model) leads us to a higher order theory without ghosts in the spectrum, the differences will be explained in the master action context. The spin-2 higher order model, is the linearized version of the new massive gravity suggested in [6].

We start with the Fierz-Pauli [15] theory, which describes in $D = 2+1$ a parity doublet of massive particles of helicities $+2$ and -2, see [10] again for a simple proof. Introducing

[1] If we linearize the Proca theory by introducing an auxiliary vector field we do derive a physically consistent gauge model dual to the Proca theory which is the $D = 2+1$ version of the Kalb-Ramond theory.

a source term we can write this theory as follows:

$$S_{\rm FP}[j] = \int d^3x \left[\frac{1}{2}T_{\mu\nu}(h)T^{\nu\mu}(h) - \frac{1}{4}T^2(h) - \frac{m^2}{2}(h_{\mu\nu}h^{\nu\mu} - h^2) + j_{\mu\nu}h^{\mu\nu}\right] \tag{2.7}$$

$$= \int d^3x \left[\frac{1}{2}\left(\sqrt{-g}R\right)_{\rm hh} - \frac{m^2}{2}(h_{\mu\nu}h^{\nu\mu} - h^2) + j_{\mu\nu}h^{\mu\nu}\right] \tag{2.8}$$

where $(\sqrt{-g}R)_{\rm hh}$ stands for the quadratic truncation of the Einstein-Hilbert action in the Dreibein fluctuations about a flat background ($e_{\alpha\beta} = \eta_{\alpha\beta} + h_{\alpha\beta}$) and

$$T_{\mu\nu}(h) \equiv \epsilon_{\mu\alpha\beta}\partial^\alpha h^\beta_{\ \nu} = -E_{\mu\beta}h^\beta_{\ \nu} \quad , \tag{2.9}$$

$$E_{\mu\beta} \equiv \epsilon_{\mu\beta\delta}\partial^\delta \tag{2.10}$$

It is important to mention that throughout this work we use second rank tensor fields, like $h_{\alpha\beta}$ in (2.7), with no symmetry in their indices. Symmetric and antisymmetric combinations will be denoted respectively by: $h_{(\alpha\beta)} \equiv (h_{\alpha\beta} + h_{\beta\alpha})/2$ and $h_{[\alpha\beta]} \equiv (h_{\alpha\beta} - h_{\beta\alpha})/2$.

The Fierz-Pauli action leads to the following equations of motion (at vanishing sources)

$$E^{\mu\alpha}E^{\nu\beta}h_{(\alpha\beta)} = m^2\,(h^{\nu\mu} - \eta^{\nu\mu}\,h) \tag{2.11}$$

from which one can derive all the required constraints to describe a spin-2 particle in $D = 2+1$:

$$h = h^\mu_\mu = 0 \tag{2.12}$$

$$h_{[\alpha\beta]} = 0 \tag{2.13}$$

$$\partial^\alpha h_{\alpha\beta} = 0 = \partial^\beta h_{\alpha\beta} \tag{2.14}$$

as well as the Klein-Gordon equation $\left(\Box - m^2\right)h_{\alpha\beta} = 0$.

Regarding the NGE procedure it is important to note that the Einstein-Hilbert action $(\sqrt{-g}R)_{\rm hh}$ is invariant under the local gauge symmetries:

$$\delta_G h_{\mu\nu} = \partial_\mu\xi_\nu + \epsilon_{\mu\nu\alpha}\Lambda^\alpha \tag{2.15}$$

which are broken by the Fierz-Pauli mass term. In order to embed these symmetries in a new model, we calculate the Euler tensor from $S_{\rm FP}[j]$:

$$M^{\mu\nu} = \frac{\delta S_{\rm FP}[j]}{\delta h_{\mu\nu}} = E^{\beta\nu}T^\nu_{\ \beta} - \frac{1}{2}E^{\nu\mu}T - m^2(h^{\nu\mu} - \eta^{\mu\nu}h) + j^{\mu\nu} \tag{2.16}$$

Then, we propose the following first iterated action S_1 by using an auxiliary tensor field $a_{\mu\nu}$ such that

$$\delta_G a_{\mu\nu} = -\delta_G h_{\mu\nu} \tag{2.17}$$

$$S_1 = \int d^3x\,\left(\mathcal{L}_{FP} + a_{\mu\nu}M^{\mu\nu} + j_{\mu\nu}h^{\mu\nu}\right) \tag{2.18}$$

Following the same steps of the previous subsection, it is easy to prove that the action below, is invariant under the gauge transformations (2.15), (2.17):

$$S_2 = \int d^3x\left(\mathcal{L}_{FP} + j_{\mu\nu}h^{\mu\nu} + a_{\mu\nu}M^{\mu\nu} - \frac{m^2}{2}(a_{\mu\nu}a^{\nu\mu} - a^2)\right) \tag{2.19}$$

Getting rid of the auxiliary fields by means of their algebraic equations of motion we obtain

$$\mathcal{L}_{\rm inv.} = \mathcal{L}_{FP} + j_{\mu\nu}h^{\mu\nu} + \frac{1}{2m^2}M_{\mu\nu}M^{\nu\mu} - \frac{1}{4m^2}M^2 \tag{2.20}$$

$$= -\frac{1}{2}T_{\mu\nu}T^{\nu\mu} + \frac{1}{4}T^2 + \frac{1}{4m^2}h_{(\rho\sigma)}\Box^2(2\theta^{\rho\nu}\theta^{\sigma\mu} - \theta^{\rho\sigma}\theta^{\mu\nu})h_{(\mu\nu)} + j_{\mu\nu}H^{\mu\nu}(h) \tag{2.21}$$

where we have neglected quadratic terms in the sources which are not important for our purposes and

$$H^{\mu\nu}(h) = \frac{1}{m^2}\left[-E^{\beta\mu}E^{\nu\alpha} + \frac{1}{2}E^{\nu\mu}E^{\beta\alpha} + \frac{\eta^{\mu\nu}}{2}E^{\beta\gamma}E_{\gamma}{}^{\alpha}\right]h_{\alpha\beta} \tag{2.22}$$

The action (2.21), at vanishing sources, corresponds exactly to the quadratic truncation of the new massive gravity recently proposed [6] up to an overall 1/2 factor, i.e.,

$$\mathcal{L}_{\rm inv.} = \mathcal{L}_{BHT}(j) = \left[-\frac{\sqrt{-g}}{2}R + \frac{1}{2m^2}\left(R_{\mu\nu}R^{\mu\nu} - \frac{3}{8}R^2\right)\right]_{\rm hh} + j_{\mu\nu}H^{\mu\nu}(h) \tag{2.23}$$

The action (2.23) is invariant under the local symmetries (2.15) as required. From the linear terms in the sources of (2.7) and (2.23) we have the dual map

$$h^{\mu\nu} \leftrightarrow H^{\mu\nu}(h) \tag{2.24}$$

The quadratic terms in the Euler tensors in (2.20) assure once again that the equations of motion of the Fierz-Pauli theory (at vanishing sources) are embedded in the BHT equations of motion which are, at $j_{\mu\nu} = 0$,

$$\Box^2(2\theta^{\rho\nu}\theta^{\sigma\mu} - \theta^{\rho\sigma}\theta^{\mu\nu})h_{(\mu\nu)} = 2m^2\left(E^{\rho\mu}E^{\sigma\nu}\right)h_{(\mu\nu)} \quad . \tag{2.25}$$

In fact we can rewrite (2.25) in the form (2.11) by noting that from (2.22) we have $m^2(H^{\mu\nu}(h) - \eta^{\mu\nu}H(h)) = E^{\rho\mu}E^{\sigma\nu}h_{(\rho\sigma)}$ and applying the operator $E^{\rho\mu}E^{\sigma\nu}$ on (2.22) we can verify $E^{\rho\mu}E^{\sigma\nu}H_{\mu\nu}(h) = \Box^2\left(2\theta^{\beta\rho}\theta^{\alpha\sigma} - \theta^{\rho\sigma}\theta^{\beta\alpha}\right)h_{(\alpha\beta)}/2m^2$. Then, (2.25) implies $E^{\mu\alpha}E^{\nu\beta}H_{\alpha\beta} = m^2\left[H^{(\nu\mu)}(h) - \eta^{\nu\mu}H(h)\right]$ which, compare with (2.11), confirms the dual map (2.24) at classical level.

From the remarks of the previous subsection one might wonder whether the higher derivative BHT theory contains ghosts introduced by the NGE procedure. As shown in [16] this is not the case. A simple demonstration is based on the following master action [6] which parallels (2.6):

$$S_M\left[h,\tilde{h}\right] = \int d^3x\left[\frac{1}{2}\left(\sqrt{-g}R\right)_{\rm hh} - \frac{m^2}{2}(h_{\mu\nu}h^{\nu\mu} - h^2) - \frac{1}{2}\left(\sqrt{-g}R\right)_{h-\tilde{h},h-\tilde{h}}\right] \tag{2.26}$$

On one hand, the additional mixing term of the Einstein-Hilbert type cancels out the first term of (2.26) such that the integration over $h_{\mu\nu}$ becomes Gaussian which gives rise exactly to the BHT theory (2.23) with $h_{\mu\nu}$ substituted by the dual field $\tilde{h}_{\mu\nu}$. On the other hand, if instead of integrating over $h_{\mu\nu}$ we shift $\tilde{h}_{\mu\nu} \to \tilde{h}_{\mu\nu} + h_{\mu\nu}$ the "wrong" sign Einstein-Hilbert term decouples from the Fierz-Pauli theory. Since, contrary to the Maxwell term in the spin-1 case, the Einstein-Hilbert term has no particle content, the spectrum of the BHT model must be the same of the Fierz-Pauli theory which explains the success of the NGE procedure in the spin-2 case.

JHEP09(2009)011

3 Gauge embedment of parity singlets of spin-2

3.1 Embedding $S_{\rm SD}^{(1)}$ in $S_{\rm SD}^{(2)}$

In this section our starting point is the first order self-dual model of [7] which describes a massive particle of helicity +2 in $D = 2+1$. Introducing a source term for future purposes we have:

$$S_{\rm SD}^{(1)}[j] = \int d^3x \left[\frac{m}{2}\epsilon^{\mu\nu\lambda} f_\mu{}^\alpha \partial_\nu f_{\lambda\alpha} - \frac{m^2}{2}(f_{\mu\nu}f^{\nu\mu} - f^2) + f_{\mu\nu}j^{\mu\nu}\right] \tag{3.1}$$

The equations of motion of (3.1) in the absence of sources,

$$E_\mu^\lambda f_{\lambda\alpha} = m\,(\eta_{\mu\alpha}\, f - f_{\alpha\mu}) \qquad , \tag{3.2}$$

also lead to the constraints (2.12), (2.13) and (2.14) and the Klein-Gordon equation $(\Box - m^2)\, f_{\alpha\beta} = 0$. From (3.2) we have the helicity equation $\left(J^\mu P_\mu/\sqrt{-P^2}+2\right)^{\alpha\beta\gamma\delta} f_{\gamma\delta} = 0$, with $2^{\alpha\beta\gamma\delta} = 2\left(\delta^{\alpha\gamma}\delta^{\beta\delta}+\delta^{\alpha\delta}\delta^{\beta\gamma}\right)$, $P_\mu = -i\partial_\mu$ and $(J^\mu)^{\alpha\beta\gamma\delta} = i\,\left(\eta^{\alpha\gamma}\epsilon^{\beta\mu\delta}+\eta^{\beta\gamma}\epsilon^{\alpha\mu\delta}+\eta^{\alpha\delta}\epsilon^{\beta\mu\gamma}+\eta^{\beta\delta}\epsilon^{\alpha\mu\gamma}\right)/2$, see [17], which assures that we are dealing with a parity singlet of helicity +2.

The local symmetry

$$\delta_\xi f_{\mu\nu} = \partial_\mu \xi_\nu \tag{3.3}$$

of the first term in (3.1) is broken by the Fierz-Pauli mass term and the source term. However, through the Noether gauge embedment procedure, we can recover it. Repeating the procedure of last section, we begin by computing the Euler tensor:

$$M^{\beta\gamma} = \frac{\delta S_{\rm SD}^{(1)}[j]}{\delta f_{\beta\gamma}} = -mE^{\beta\lambda} f_\lambda{}^\gamma - m^2(f^{\gamma\beta} - \eta^{\beta\gamma} f) + j^{\beta\gamma} \tag{3.4}$$

With the help of an auxiliary field which satisfies $\delta_\xi a_{\beta\gamma} = -\partial_\beta \xi_\gamma$ we can define a first-iterated action

$$\delta_\xi S^{(1)} = \delta_\xi \int d^3x \left(\mathcal{L}_{SD}^{(1)} + a_{\beta\gamma}M^{\beta\gamma}\right) = \int d^3x\, a_{\beta\gamma}\delta_\xi M^{\beta\gamma} = \delta_\xi \int d^3x\, \frac{m^2}{2}(a_{\beta\gamma}a^{\gamma\beta} - a^2), \tag{3.5}$$

where we have used $\delta_\xi a_{\mu\nu} = -\delta_\xi f_{\mu\nu}$. Thus, we can obtain the gauge invariant model:

$$\mathcal{L}_2 = \mathcal{L}_{SD}^{(1)} - a_{\beta\gamma}M^{\beta\gamma} - \frac{m^2}{2}(a_{\beta\gamma}a^{\gamma\beta} - a^2) \tag{3.6}$$

Solving the equations of motion for $a_{\beta\gamma}$ and replacing the solutions we find, after dropping quadratic terms in the sources,

$$\mathcal{L}_{SD}^{(2)} = \mathcal{L}_{SD}^{(1)} + f_{\mu\nu}j^{\mu\nu} + \frac{1}{2m^2}M_{\nu\mu}M^{\mu\nu} - \frac{1}{4m^2}M^2 \tag{3.7}$$

$$= \frac{1}{2}T_{\mu\nu}(f)T^{\nu\mu}(f) - \frac{1}{4}T^2(f) - \frac{m}{2}f_{\mu\nu}T^{\mu\nu}(f) + j_{\mu\nu}F^{\mu\nu}(f), \tag{3.8}$$

$$= \frac{1}{2}\left(\sqrt{-g}R\right)_{\rm ff} - \frac{m}{2}\epsilon^{\mu\alpha\beta} f_{\mu\nu}\partial_\alpha f_\beta{}^\nu + j_{\mu\nu}F^{\mu\nu}(f), \tag{3.9}$$

where $e_{\alpha\beta} = \eta_{\alpha\beta} + f_{\alpha\beta}$ and

$$F^{\mu\nu}(f) = \frac{1}{m}\left[T^{\nu\mu}(f) - \frac{\eta^{\mu\nu}}{2}T(f)\right] \quad , \tag{3.10}$$

Note that the last two terms in (3.7) are quadratic in the Euler tensor which guarantees again the embedment of the equations of motion of $\mathcal{L}_{SD}^{(1)}$ in the second-order model $\mathcal{L}_{SD}^{(2)}$ which has appeared before in [8]. Comparing the terms linear in the sources in (3.1) and (3.9) we arrive at the dual map between $\mathcal{L}_{SD}^{(1)}$ and $\mathcal{L}_{SD}^{(2)}$:

$$f^{\mu\nu} \leftrightarrow F^{\mu\nu}(f) \tag{3.11}$$

Indeed, minimizing (3.8) at vanishing sources we find:

$$E_{\mu\alpha}\left(T_{\nu}^{\alpha}(f) - \eta_{\nu}^{\alpha}\frac{T(f)}{2}\right) = -m\,T_{\mu\nu}(f) \quad , \tag{3.12}$$

which can be recast as:

$$E_{\mu}^{\lambda}F_{\lambda\alpha} = m\left(\eta_{\mu\alpha}F - F_{\alpha\mu}\right) \quad . \tag{3.13}$$

Comparing with (3.2) we confirm the dual map (3.11) at classical level. The same map[2] holds at quantum level up to contact terms in the correlation functions. Since contact terms have no poles, the particle content of $\mathcal{L}_{SD}^{(1)}$ and $\mathcal{L}_{SD}^{(2)}$ coincide, namely, one massive mode of helicity $+2$. From the master action point of view this is a consequence of using a first-order Chern-Simons term (CS_1), which has no particle content, as a mixing term in going from $\mathcal{L}_{SD}^{(1)}$ to $\mathcal{L}_{SD}^{(2)}$ [9].

3.2 Embedding $S_{\rm SD}^{(2)}$ in $S_{\rm SD}^{(3)}$

It turns out that the Einstein-Hilbert term in (3.9) depends only upon the symmetric combination $f_{(\mu\nu)}$, therefore it is invariant under the local symmetry:

$$\delta_\Gamma f_{\mu\nu} = \epsilon_{\mu\nu\alpha}\Gamma^{\alpha} \quad , \tag{3.14}$$

which is broken by the first-order Chern-Simons mass term in (3.9). This suggests another round of the NGE procedure. The Euler tensor from $S_{SD_2}^{(2)}$ is given by

$$M^{\mu\nu} = \frac{\delta S_2^{(2)}}{\delta f_{\mu\nu}} = E^{\beta\mu}T^{\nu}{}_{\beta} - \frac{1}{2}E^{\nu\mu}T - mT^{\mu\nu} + G^{\mu\nu}(j). \tag{3.15}$$

where we have defined

$$G^{\mu\nu}(j) = \frac{E^{\lambda\mu}j^{\nu}{}_{\lambda}}{m} - \frac{E^{\nu\mu}j}{2m}. \tag{3.16}$$

Again, with the help of an auxiliary field $a_{\beta\gamma}$ such that

$$\delta_\Gamma a_{\mu\nu} = -\epsilon_{\mu\nu\delta}\Gamma^{\delta} = -\delta_\Gamma f_{\mu\nu} \tag{3.17}$$

[2]There is a mistake in the definition of the dual $F_{\mu\nu}$ on the right handed side of formula (19) of [9] where $T_{\mu\nu}$ should be replaced by $T_{\nu\mu}$.

we can write the first iterated action:

$$S_1 = \int d^3\, x \left[\mathcal{L}^{(2)}_{SD} + f_{\mu\nu} G^{\mu\nu}(j) + a_{\mu\nu} M^{\mu\nu} + \mathcal{O}(j^2)\right] \tag{3.18}$$

such that

$$\delta_\Gamma S_1 = \delta_\Gamma \left(-\int d^3\, x \frac{m}{2}(a_{\mu\nu} E^{\mu\beta} a_\beta{}^\nu)\right) \tag{3.19}$$

So we derive

$$\mathcal{L}_2 = \mathcal{L}^{(2)}_{SD} + a_{\mu\nu} M^{\mu\nu} + \frac{m}{2}(a_{\mu\nu} E^{\mu\beta} a_\beta{}^\nu) + f_{\mu\nu} G^{\mu\nu}(j) + \mathcal{O}(j^2), \tag{3.20}$$

which is invariant under (3.14) altogether with (3.17). Although the equations of motion of the auxiliary fields are not algebraic as in the last subsection, they can still be eliminated in a trivial way leaving us with a local gauge invariant action. For this aim, note that the Euler tensor can be written as

$$M^{\mu\nu} = E^{\mu\beta}\left(-T^\nu{}_\beta + \frac{\eta^\nu_\beta}{2} T + m f_\beta{}^\nu - \frac{j^\nu{}_\beta}{m} + \frac{\eta^\nu_\beta j}{2m}\right) \equiv E^{\mu\beta} b_\beta{}^\nu \tag{3.21}$$

Now we can decouple the auxiliary fields by using:

$$a_{\mu\nu} E^{\mu\beta} b^\nu_\beta + \frac{m}{2}(a_{\mu\nu} E^{\mu\beta} a_\beta{}^\nu) = -\frac{1}{2m} b_{\mu\nu} E^{\mu\beta} b^\nu_\beta + \frac{m}{2}(\tilde{a}_{\mu\nu} E^{\mu\beta} \tilde{a}_\beta{}^\nu) \quad , \tag{3.22}$$

Where $\tilde{a}_{\mu\nu} = a_{\mu\nu} + b_{\mu\nu}/m$. Neglecting the last term in (3.22) which has no particle content, we obtain the invariant Lagrangian density:

$$\mathcal{L}_{\text{inv.}} = \mathcal{L}^{(2)}_{SD} - \frac{1}{2m} b_{\mu\nu} M^{\mu\nu} + f_{\mu\nu} G^{\mu\nu}(j) \tag{3.23}$$

Once again we have neglected quadratic terms in the sources. Although (3.23) is linear in the Euler tensor, due to (3.21) we have the general variation:

$$\delta S_{\text{inv.}} = \int d^3x\, M^{\mu\nu}\left(\delta f_{\mu\nu} - \frac{1}{m}\delta b_{\mu\nu}\right) \tag{3.24}$$

Consequently, the equations of motion of $S^{(2)}_{\text{SD}}$, $M^{\mu\nu} = 0$, are embedded in the equations of motion of $S_{\text{inv.}}$. The action $S_{\text{inv.}}$ is of third-order and can be rewritten, dropping terms $\mathcal{O}(j^2)$, as

$$\mathcal{L}_{\text{inv.}} \equiv \mathcal{L}^{(3)}_{SD} = -\frac{1}{2m} f_{\alpha\mu}(\Box\theta^{\alpha\gamma} E^{\beta\mu} - \Box\theta^{\alpha\mu} E^{\beta\gamma}) f_{\gamma\beta} - \frac{1}{2} T_{\mu\nu} T^{\nu\mu} + \frac{1}{4} T^2 + j_{\mu\nu} \tilde{F}^{\mu\nu}(f) \tag{3.25}$$

$$= -\frac{1}{8m}\left\{\epsilon^{\mu\nu\rho}\Gamma^\epsilon_{\mu\gamma}\left[\partial_\nu \Gamma^\gamma_{\epsilon\rho} + (2/3)\Gamma^\gamma_{\nu\delta}\Gamma^\delta_{\rho\epsilon}\right]\right\}_{\text{ff}} - \frac{1}{2}\left(\sqrt{-g}R\right)_{\text{ff}} + j_{\mu\nu}\tilde{F}^{\mu\nu}(f) \tag{3.26}$$

Note the change of the sign of the Einstein-Hilbert term. This is similar to the change of the sign of the Maxwell term in going from (2.1) to (2.5). The theory $\mathcal{L}^{(3)}_{SD}$ corresponds to the quadratic truncation of the topologically massive gravity of [1]. Above, we have defined

$$\tilde{F}^{\alpha\beta}(f) = \frac{E^{\alpha\gamma} E^{\beta\lambda} f_{(\gamma\lambda)}}{m^2} \tag{3.27}$$

Comparing (3.1) and (3.26) we find the dual map

$$f_{\alpha\beta} \leftrightarrow \tilde{F}_{\alpha\beta} \quad . \tag{3.28}$$

Once again, the dual map (3.28) holds at classical and quantum level (up to contact terms), see [9], which assures the spectrum equivalence between $S^{(3)}_{\text{SD}}$ and $S^{(2)}_{\text{SD}}$.

JHEP09(2009)011

3.3 New self-dual model for spin-2 particles

Only the symmetric combination $f_{(\mu\nu)}$ appears in $S_{\rm SD}^{(3)}$, explicitly,

$$S_{\rm SD}^{(3)}[j] = \int d^3x \left[-\frac{1}{2m} f_{(\lambda\mu)} \Box \theta^{\lambda\alpha} E^{\mu\delta} f_{(\alpha\delta)} - \frac{1}{2} f_{(\lambda\mu)} E^{\lambda\delta} E^{\mu\alpha} f_{(\alpha\delta)} + j_{\lambda\mu} \tilde{F}^{\lambda\mu}(f) \right] \tag{3.29}$$

Once more the highest derivative term of the action contains an extra local symmetry not shared by the remaining terms. Namely, the first term of $S_{\rm SD}^{(3)}$ is invariant under the linearized Weyl transformation:[3]

$$\delta_w f_{\mu\nu} = \phi\, \eta_{\mu\nu} \tag{3.30}$$

while this is not true for the Einstein-Hilbert term. By imposing this new symmetry we will arrive at yet another self-dual model for spin-2 particles in $D = 2+1$. We start with the Euler tensor

$$M^{\beta\gamma} = \frac{\delta S_{\rm SD}^{(3)}[j]}{\delta f_{\beta\gamma}} = E^{\beta\mu} E^{\gamma\nu} b_{(\mu\nu)} = M^{(\beta\gamma)} \tag{3.31}$$

where $b_{(\mu\nu)}$ is given by:

$$b_{(\mu\nu)} = -\left[f_{(\mu\nu)} + \frac{(\eta^\delta_\nu E_\mu{}^\alpha + \eta^\delta_\mu E_\nu{}^\alpha) f_{(\alpha\delta)}}{2m} + \frac{j_{(\mu\nu)}}{m^2} \right] \tag{3.32}$$

Following the same steps of last examples we end up with the action

$$S_2 = \int d^3x \left[\mathcal{L}_{SD}^{(3)} + a_{(\beta\gamma)} E^{\beta\mu} E^{\gamma\nu} b_{(\mu\nu)} - \frac{1}{2} a_{(\beta\gamma)} E^{\beta\mu} E^{\gamma\nu} a_{(\mu\nu)} \right] \quad , \tag{3.33}$$

After decoupling the auxiliary fields and neglecting a term of the Einstein-Hilbert form $(-1/2)\tilde{a}_{(\beta\gamma)} E^{\beta\mu} E^{\gamma\nu} \tilde{a}_{(\mu\nu)}$ where $\tilde{a}_{(\beta\gamma)} = a_{(\beta\gamma)} - b_{(\beta\gamma)}$, which has no propagating degree of freedom, we obtain:

$$\mathcal{L}_{SD}^{(4)} = \mathcal{L}_{SD}^{(3)} + \frac{1}{2} b_{(\beta\gamma)} E^{\beta\delta} E^{\gamma\alpha} b_{(\alpha\delta)} \tag{3.34}$$

$$= \frac{1}{4m^2} f_{(\rho\sigma)} (2\Box^2 \theta^{\rho\nu} \theta^{\sigma\mu} - \Box^2 \theta^{\rho\sigma} \theta^{\mu\nu}) f_{(\mu\nu)} + \frac{1}{2m} f_{(\lambda\mu)} \Box \theta^{\lambda\alpha} E^{\mu\delta} f_{(\alpha\delta)} \tag{3.35}$$

$$- \frac{j_{(\alpha\delta)} E^{\rho\alpha} \Box \theta^{\delta\sigma} f_{(\sigma\rho)}}{m^3} \tag{3.36}$$

This is a new self-dual model for particles of helicity $+2$ (or -2 depending on the sign of the third-order term). It corresponds to the sum of a third order gravitational Chern-Simons term $\mathcal{L}_{CS3} \equiv \epsilon^{\mu\nu\rho} \Gamma^\epsilon_{\mu\gamma} \left[\partial_\nu \Gamma^\gamma_{\epsilon\rho} + (2/3) \Gamma^\gamma_{\nu\delta} \Gamma^\delta_{\rho\epsilon} \right]$ and the fine tuned curvature square term of [6] at linearized level with appropriate coefficients, i.e.,

$$\mathcal{L}_{SD}^{(4)} = \frac{1}{2m^2} \left(R^{\mu\nu} R_{\mu\nu} - \frac{3}{8} R^2 \right)_{\rm ff} + \frac{1}{8m} \left\{ \epsilon^{\mu\nu\rho} \Gamma^\epsilon_{\mu\gamma} \left[\partial_\nu \Gamma^\gamma_{\epsilon\rho} + (2/3) \Gamma^\gamma_{\nu\delta} \Gamma^\delta_{\rho\epsilon} \right] \right\}_{\rm ff} + j_{(\alpha\beta)} G^{\alpha\beta}(f) \tag{3.37}$$

[3] The third order gravitational Chern-Simons (CS_3) term is invariant under Weyl transformations $\delta_w g_{\mu\nu} = 2\phi\, g_{\mu\nu}$ which reduce to $\delta_w f_{\mu\nu} = \phi\, \eta_{\mu\nu}$ when we truncate CS_3 to quadratic terms about a flat background

where

$$G_{\alpha\beta}(f) = -\frac{\Box}{2m^3}\left[E^{\rho}_{\alpha}\theta^{\delta}_{\beta} + E^{\rho}_{\beta}\theta^{\delta}_{\alpha}\right] f_{(\rho\delta)} \tag{3.38}$$

The new model is invariant under all local symmetries (3.3), (3.14) and (3.30). Since both quartic- and third-order terms of (3.37) are invariant by the same set of gauge symmetries, the NGE procedure naturally terminates. Comparing (3.1) and (3.37) we have the duality between $S^{(1)}_{\rm SD}$ and $S^{(4)}_{\rm SD}$ established by the dual map:

$$f_{\mu\nu} \leftrightarrow G_{\mu\nu}(f) \tag{3.39}$$

By using the identities $E_{\nu\mu}\theta^{\mu\delta} = E^{\delta}_{\nu}$, $E_{\nu\mu}E^{\mu\delta} = \Box\theta^{\delta}_{\nu}$ and $E^{\nu\alpha}E^{\lambda\delta} = \left(\theta^{\nu\delta}\theta^{\alpha\lambda} - \theta^{\nu\lambda}\theta^{\alpha\delta}\right)$ it is easy to derive from (3.38) that $E^{\lambda}_{\gamma}G^{\mu\gamma} = -(\Box^2/2m^3)\left(\theta^{\lambda\alpha}\theta^{\mu\beta} + \theta^{\mu\alpha}\theta^{\lambda\beta} - \theta^{\lambda\mu}\theta^{\alpha\beta}\right) f_{(\alpha\beta)}$. Consequently, the equations of motion of $S^{(4)}_{\rm SD}$:

$$\Box^2\left(\theta^{\lambda\alpha}\theta^{\mu\beta} + \theta^{\mu\alpha}\theta^{\lambda\beta} - \theta^{\lambda\mu}\theta^{\alpha\beta}\right) f_{(\alpha\beta)} = \Box\left(\theta^{\lambda\alpha}E^{\mu\beta} + \theta^{\mu\alpha}E^{\lambda\beta}\right) f_{(\alpha\beta)} \tag{3.40}$$

can be recast as

$$E^{\nu}_{\mu}G^{\mu\lambda} = G^{\nu\lambda} \tag{3.41}$$

which is exactly of the form (3.2) if we note, see (3.38), the identity $G^{\mu}_{\mu} = 0$. Consequently, the dual map (3.39) between $S^{(1)}_{\rm SD}$ and $S^{(4)}_{\rm SD}$ is verified at classical level. In particular, if we apply the operator E^{α}_{ν} on (3.41) and use(3.41) again we obtain the Klein-Gordon equation $\left(\Box - m^2\right) G_{\alpha\beta} = 0$. It is remarkable that *all* necessary constraints to describe a spin-2 massive particle, i.e., $G^{\mu}_{\mu} = 0 = G_{[\mu\nu]}$, $\partial^{\mu}G_{\mu\nu} = 0 = \partial^{\nu}G_{\mu\nu}$ follow now from trivial identities instead of dynamic equations differently from the other three self-dual models. In this sense the $S^{(4)}_{\rm SD}$ model is the most natural description of spin-2 parity singlets in $D = 2+1$.

Regarding the particle content of the $S^{(4)}_{\rm SD}$ model at quantum level, we turn again to a master action:

$$S_M[f,\tilde{f}] = \int d^3x\,\frac{1}{2}\left[\frac{1}{m}\left(\mathcal{L}_{CS3}\right)_{\rm ff} - \left(\sqrt{-g}R\right)_{\rm ff} - \frac{1}{m}\left(\mathcal{L}_{CS3}\right)_{f-\tilde{f},f-\tilde{f}}\right] \tag{3.42}$$

Using the notation of [9] we can rewrite (3.42) as follows:[4]

$$\begin{aligned} S_M[f,\tilde{f}] &= \frac{1}{4}\int\left[-\frac{\Omega(f)\cdot d\Omega(f)}{m} + f\cdot d\Omega(f) + \frac{\Omega(f-\tilde{f})\cdot d\Omega(f-\tilde{f})}{m}\right] && (3.43)\\ &= \frac{1}{4}\int\left[\frac{\Omega(\tilde{f})\cdot d\Omega(\tilde{f})}{m} - \frac{2\,\Omega(\tilde{f})\cdot d\Omega(f)}{m} + f\cdot d\Omega(f)\right] && (3.44)\\ &= \frac{1}{4}\int\left[-\frac{\Omega(\tilde{f})\cdot d\Omega(\Omega(\tilde{f}))}{m^2} + \frac{\Omega(\tilde{f})\cdot d\Omega(\tilde{f})}{m} + \left(f - \frac{\Omega(\tilde{f})}{m}\right)\cdot d\Omega\left(f - \frac{\Omega(\tilde{f})}{m}\right)\right] && (3.45) \end{aligned}$$

After the shift $f \to f + \Omega(\tilde{f})/m$ the first two terms of (3.45) correspond exactly to the new $S^{(4)}_{\rm SD}$ self-dual model as function of $\tilde{f}_{\mu\nu}$ while the last term is a pure Einstein-Hilbert action

[4] In [9] we have defined $\int h\cdot g \equiv \int d^3x\, h^{\mu\nu}\epsilon_{\mu}^{\ \alpha\beta}\partial_{\alpha}g_{\beta\nu}$ and used $\Omega^{\alpha}{}_{\lambda}(f) = -\epsilon^{\alpha\beta\gamma}[\partial_{\lambda}f_{\gamma\beta} + 2\,\partial_{\gamma}f_{(\lambda\beta)}]$ as defined in [8].

depending only on $f_{\mu\nu}$. So the particle content of the master action (3.42) is the same of $S^{(4)}_{\rm SD}$. On the other hand, if we start from (3.43) and shift $\tilde{f}_{\mu\nu} \to \tilde{f}_{\mu\nu} + f_{\mu\nu}$ we obtain the $S^{(3)}_{\rm SD}[f]$ model and a pure (decoupled) linearized gravitational Chern-Simons term with no particle content [18]. Therefore, it is clear that $S^{(4)}_{\rm SD}$ and $S^{(3)}_{\rm SD}$ share the same spectrum, i.e., one massive physical particle of helicity +2. This can be confirmed by a calculation of the sign of the imaginary part of the residues at the poles of the propagator of $S^{(4)}_{\rm SD}$ when saturated with conserved and traceless (as required by the linearized Weyl symmetry) sources. In fact, there are two poles in the propagator of the $S^{(4)}_{\rm SD}$ model, one massive and one massless (ghost-like). It turns out that the traceless condition on the sources get rid of a ghost-like massless pole and we are left with one physical massive pole [19].

4 Conclusion and comments

In the second section we have shown that the (linearized) BHT model can be obtained from the Fierz-Pauli theory via Noether embedment. In principle, the same procedure applies in higher dimensions however, the Einstein-Hilbert action becomes dynamical for $D > 3$ and by the arguments given here we expect that the embedment would lead to a massless ghost. In other words, for $D > 3$ the NGE of spin-2 massive particles is similar to the spin-1 case where we have obtained a ("wrong" sign) Maxwell-Podolsky theory.

The section 3 contains a natural chain of Noether gauge embedment: $S^{(1)}_{\rm SD} \to S^{(2)}_{\rm SD} \to S^{(3)}_{\rm SD} \to S^{(4)}_{\rm SD}$. All terms of the $S^{(4)}_{\rm SD}$ model have the same local symmetry, so the embedment terminates at the $S^{(4)}_{\rm SD}$. This is similar to the spin-1 case where both Maxwell and firs-order Chern-Simons terms are invariant under the same gauge transformations, so the embedment of the first-order model of [2] terminates after one round at the MCS theory.

It is interesting to remark that only in the model $S^{(4)}_{\rm SD}$ the necessary constraints to describe a spin-2 particle are identically (non-dynamically) satisfied which make us believe that $S^{(4)}_{\rm SD}$ is the most natural description of spin-2 parity singlets in $D = 2 + 1$, just like the MCS theory automatically incorporates the transverse condition $\partial_\mu F^\mu = 0$, where $F_\mu = \epsilon_{\mu\nu\alpha}\partial^\nu A^\alpha/m$ is the dual of the self-dual field f_μ of [2]. Quite surprisingly, the $S^{(4)}_{\rm SD}$ model, which contains only third- and fourth-order terms, is spectrally equivalent to the other lower-order self-dual models. From the master action point of view this follows from the triviality (no particle content) of the linearized third-order gravitational Chern-Simons term (CS_3). In fact, from this standpoint, the existence of the dual theories $S^{(2)}_{\rm SD}$, $S^{(3)}_{\rm SD}$ and $S^{(4)}_{\rm SD}$ follows from the trivial cohomology of the differential operators which appear in the CS_1, linearized Einstein-Hilbert and linearized CS_3 terms. There seems to be a one-to-one correspondence between differential operators of trivial cohomology and dual theories. This is also true in the spin-1 case where the first-order topological Chern-Simons term (CS_1) is apparently the only one which could be used in the master action approach to generate a dual theory to the first-order self-dual model of [2], in this case one obtains the Maxwell-Chern-Simons theory of [1].

Finally, since $S^{(4)}_{\rm SD}$ may be interpreted as the linearized version of the model $\mathcal{L}^{\pm}_{HDTMG} = \pm\epsilon^{\mu\nu\rho}\Gamma^{\epsilon}_{\mu\gamma}\left[\partial_\nu\Gamma^{\gamma}_{\epsilon\rho} + (2/3)\Gamma^{\gamma}_{\nu\delta}\Gamma^{\delta}_{\rho\epsilon}\right]/8m + \sqrt{-g}\left[R_{\mu\nu}R^{\nu\mu} - 3\,R^2/8\right]/2m^2$, one might consider it as a toy model for a massive gravitational theory despite the absence of the Einstein-Hilbert term.

Note added. After uploading our work we have been informed of the preprint [20] where the $S_{\rm SD}^{(4)}$ model also appears.

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